A Constant in the Theory of Trigonometric Series

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It is known [1, vol. 1, p. 191] that there is a number α_0, $0 < \alpha_0 < 1$, such that for each $\alpha \geqq \alpha_0$ the partial sums of $\sum_{n=1}^{\infty} n^{-\alpha} \cos nx$ are uniformly bounded below, whereas for $\alpha < \alpha_0$ they are not; α_0 is the root of

$$F(\alpha) \equiv \int_0^{3\pi/2} u^{-\alpha} \cos u \, du = 0.$$

The computation from an ALGOL program on the IBM 709 at Northwestern University gives the following results:

α	$F(\alpha)$
0.30480	-0.12468407 (10^{-3})
0.30481	-0.88087283 (10^{-4})
0.30482	-0.51491894 (10^{-4})
0.30483	-0.14883466 (10^{-4})
0.30484	0.21690037 (10^{-4})
0.30485	0.58313366 (10^{-4})
0.30486	0.94888266 (10^{-4})
0.30487	0.13149530 (10^{-3}).

Hence $0.30483 < \alpha_0 < 0.30484$.

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1. A. ZYGMUND, *Trigonometric Series*, 2nd ed., Cambridge University Press, Cambridge, 1959.